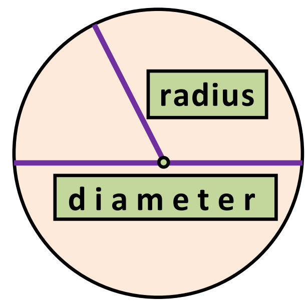
CIRCLES – ARITHMETIC

❑ RADIUS AND DIAMETER



The distance from the center of a circle to any point on the circle is called its ***radius***. The distance from one point on a circle to another point on the circle, and passing through the center, is called its ***diameter***. It should be clear that the diameter is twice the radius:

$$\boldsymbol{d = 2r}$$

Equivalently, the radius is half the diameter $r = \frac{1}{2}d$, which can also be written as

$$\boldsymbol{r = \frac{d}{2}}$$

> The terms *radius* and *diameter* can also refer to the line segments themselves, but their lengths are much more useful.

EXAMPLE 1:

A. **The radius of a circle is 89. Find the diameter.**

Since $d = 2r$, it follows that $d = 2(89) =$ **178**.

B. **The diameter of a circle is 13. Find the radius.**

The formula $r = \frac{d}{2}$ tells us that $r = \frac{13}{2} = \mathbf{6\frac{1}{2}}$, or **6.5**, if you prefer.

C. **The diameter of a circle is $\frac{3}{5}$. Find the radius.**

$$r = \frac{d}{2} = \frac{\frac{3}{5}}{2} = \frac{3}{5} \div 2 = \frac{3}{5} \div \frac{2}{1} = \frac{3}{5} \times \frac{1}{2} = \mathbf{\frac{3}{10}}$$

Homework

1. For each problem, find the diameter if the radius is given, and find the radius if the diameter is given:

a. $r = 20$ b. $r = 7.6$ c. $r = \frac{1}{7}$ d. $r = 0.5$

e. $d = 44$ f. $d = 0.25$ g. $d = 13$ h. $d = \frac{1}{5}$

❑ DEFINITION OF π

A circle has a perimeter (the distance all the way around) just as squares and rectangles do, and we call this perimeter the ***circumference*** of the circle. The ***area*** of a circle is a measure of the region inside the circle. The formulas for circumference and area will be stated after a discussion of a very, very important number.

Choose any circle at all: tiny, mid-sized, or gigantic. When you divide the circumference by the diameter — no matter the circle — you always end up with the same number, a constant a little bigger than 3. This quotient (the circumference divided by the diameter), which is the same for all circles, is denoted by the Greek letter pi, "**π**", from the Greek word *perimetros* (though the Greeks themselves did not use the symbol π for this special number).

The modern symbol for pi — π — was first used in 1706 by William Jones.

The definition of π:

$$\pi = \frac{C}{d}$$

The circumference of any circle is a little over 3 times its diameter.

The decimal version of π has an infinite number of digits in it (with <u>no</u> repeating pattern), and therefore any decimal number we write for π will only be an approximation. The calculator gives something like 3.141592654 for π. Two useful approximations of π are 3.14 and $\frac{22}{7}$.

Since this class is getting you prepared for an algebra class, we'll stick (for now) with the exact value of π, which, of course, is simply written $\boldsymbol{\pi}$.

> Even though π has been calculated to about 202 trillion digits, according to NASA, 40 digits of π are all that is required to measure the distance around the universe to an accuracy equal to the diameter of a hydrogen atom. That's pretty darned accurate!
>
> See Digits of Pi

❑ *CIRCUMFERENCE AND AREA*

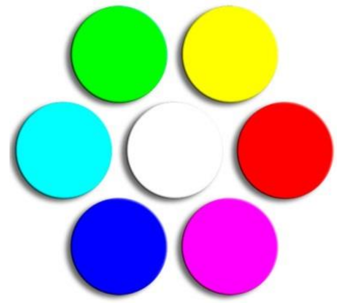
The circumference of a circle can be found by multiplying 2 times π times the radius:

$$\boldsymbol{C = 2\pi r}$$

[See if you can derive this formula yourself, using the definition of π and the fact that $d = 2r$.]

As for area, the formula is difficult to derive, so we simply state it and use it:

$$\boldsymbol{A = \pi r^2}$$



Note #1: Since the Order of Operations specifies that exponents have priority over multiplication, we note that the area formula tells us to first square the r, and then multiply by π. In other words, πr^2 means $\pi \times (r^2)$.

Note #2: Students sometimes mix up the circle formulas for circumference and area. Here's a way to remember which is which. The <u>area</u> of any geometric shape is always in <u>square</u> units, for example, square feet. And which formula, $2\pi r$ or πr^2, has the "square" in it? The πr^2 does, of course. So area is πr^2, and circumference is the one without the square in it, $2\pi r$.

Note #3: A previous teacher may have taught you that the circumference of a circle is given by the formula $C = \pi d$, instead of the formula $C = 2\pi r$ stated above. They are really the same formula, since d is the same as $2r$, so you can certainly use whichever one you like.

<u>EXAMPLE 2:</u> **The radius of a circle is 17.5. Find the circumference.**

<u>Solution:</u> Using the formula for the circumference of a circle, we proceed as follows:

$$
\begin{aligned}
C &= 2\pi r && \text{(the circumference formula)} \\
C &= 2\pi(\mathbf{17.5}) && \text{(plug in the given radius for } r\text{)} \\
C &= 2(17.5)\pi && \text{(rearrange the factors)} \\
C &= \boxed{35\pi} && \text{(multiply 2 by 17.5)}
\end{aligned}
$$

Note that this is the exact answer, because the symbol π is in the final answer. An approximation could easily be found by using 3.14 for π, and then multiplying 35 by 3.14 to get **109.9**.

EXAMPLE 3: **The radius of a circle is 17. Find the area.**

Solution: The relevant formula is $A = \pi r^2$. Notice that only the r is being squared in this formula, since the Order of Operations specifies that exponents have priority over multiplication. We therefore get

$A = \pi r^2$ (the area of a circle formula)

$A = \pi(\mathbf{17})^2$ (plug in the given radius for r)

$A = \pi(289)$ (square the 17, and we're basically done)

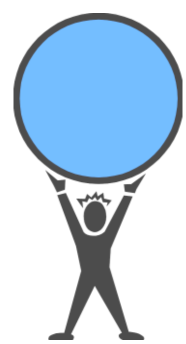
$A = \boxed{289\pi}$ (it looks prettier this way)

Homework

The Circle Formulas:

$$d = 2r \qquad r = \frac{d}{2} \qquad C = 2\pi r \qquad A = \pi r^2$$

2. Find the **circumference** of the circle with the given radius — leave your answers in exact form (that is, leave π in the answer):



a. $r = 8$ b. $r = 33.4$ c. $r = 0.07$ d. $r = 89$

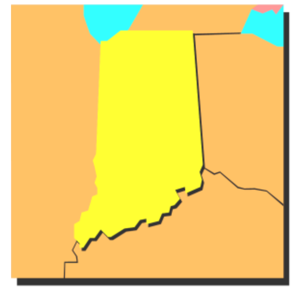
e. $r = 13$ f. $r = 0.5$ g. $r = 100$ h. $r = 77.5$

3. Find the **area** of the circle with the given radius — leave your answers in exact form (that is, leave π in the answer):

a. $r = 9$ b. $r = 100$ c. $r = 0.3$ d. $r = 3.5$

e. $r = 10$ f. $r = 2.5$ g. $r = 1$ h. $r = 0.08$

❑ THE DAY π WAS SAVED



In 1897, legislator Edwin J. Goodman of the Indiana legislature attempted to push through a law that would have indirectly declared the number 3.2 to be the exact value of π. Though Mr. Goodman's bill had unanimously passed the House of Representatives and had passed one reading in the Senate, a visitor, C.A. Waldo, a math professor from Purdue University, intervened and convinced the Senate not to pass the bill.

❑ A BIBLICAL VIEW OF π

From I Kings 7:23 comes the sentence:

> "*Then He made the molten sea, ten cubits from brim to brim, while a line of 30 cubits measured it around.*"

The word "sea" refers to a large container for holding water. "Brim to brim" refers to the diameter of 10 cubits, while the 30 cubits refers to the circumference. Thus, from this quote, we derive the value $\pi = \frac{C}{d} = \frac{30 \text{ cubits}}{10 \text{ cubits}} = \mathbf{3}$, which is quite a good estimate of π, given its great antiquity.

Review Problems

4. The radius of a circle is 2.3. Find the diameter.
5. The diameter of a circle is $\frac{4}{5}$. Find the radius.
6. The radius of a circle is 15. Find the area.
7. The radius of a circle is 12. Find the circumference.
8. Find the diameter of a circle if its radius is $\frac{3}{7}$.
9. Find the radius of a circle if its diameter is 0.28.
10. Find the circumference of a circle whose radius is 2.7.
11. Find the area of a circle whose radius is 8.8.
12. The radius of a circle is 0.8. Find the diameter.
13. The diameter of a circle is $\frac{3}{5}$. Find the radius.
14. The radius of a circle is 16. Find the area.
15. The radius of a circle is 25. Find the circumference.
16. Find the diameter of a circle if its radius is $\frac{3}{11}$.
17. Find the radius of a circle if its diameter is 0.86.
18. Find the circumference of a circle whose radius is 3.5.
19. Find the area of a circle whose radius is 2.3.
20. The radius of a circle is 0.002. Find the diameter.
21. The diameter of a circle is $\frac{9}{11}$. Find the radius.
22. The radius of a circle is 45. Find the area.
23. The radius of a circle is 126. Find the circumference.
24. Find the diameter of a circle if its radius is $\frac{4}{9}$.
25. Find the radius of a circle if its diameter is 0.27.
26. Find the circumference of a circle whose radius is 151.5.
27. Find the area of a circle whose radius is 1,000.

Solutions

1. a. $d = 40$ b. $d = 15.2$ c. $d = \frac{2}{7}$ d. $d = 1$

e. $r = 22$ f. $r = 0.125$ g. $r = 6\frac{1}{2}$ h. $r = \frac{1}{10}$

2. a. 16π b. 66.8π c. 0.14π d. 178π

e. 26π f. π g. 200π h. 155π

3. a. 81π b. $10{,}000\pi$ c. 0.09π d. 12.25π

e. 100π f. 6.25π g. π h. 0.0064π

4. 4.6 **5.** $\frac{2}{5}$ **6.** 225π **7.** 24π

8. $\frac{6}{7}$ **9.** 0.14 **10.** 5.4π **11.** 77.44π

12. 1.6 **13.** $\frac{3}{10}$ **14.** 256π **15.** 50π

16. $\frac{6}{11}$ **17.** 0.43 **18.** 7π **19.** 5.29π

20. 0.004 **21.** $\frac{9}{22}$ **22.** 2025π **23.** 252π

24. $\frac{8}{9}$ **25.** 0.135 **26.** 303π **27.** $1{,}000{,}000\pi$

"The greatest thing
you'll ever learn,
is just to love,
and be loved in return."

eden ahbez